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Packet Radio Temporary Note #27

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NOTE ON THE IMPLICATIONS OF THE POISSION PROBABILITY LAW AS APPLICABLE TO PACKET RADIO COMMUNICATIONS

1.0 SUMMARY

The objective of this note is to show that a packet radio cell in an assumed urban environment with moderate density of terminals (probably located on the tops of some buildings), omnidirectional antennas on the terminals, employing single frequency operation, and with no means of code-division discrimination is likely to be severely interference limited and will require high ERP's from a repeater to operate properly.

Once this objective is established as a valid issue, a basis is established for evaluating alternative means to mitigate the interference. These means include multiple frequency operation, use of directional antennas on at least some fixed terminals as well as on the repeaters, and use of spread spectrum or code division discrimination.

The specific conclusions of this note are:
(1) If all terminals have omnidirectional antennas, reception at a particular terminal will be interference limited at a level which may typically be about 50 dB above thermal noise.
(2) The required ERP's from a repeater assuming a 1-watt ERP from each terminal and a moderate margin for multipath may typically run 50-65 dBW depending on whether the transmission loss runs closer to 20 dB per range decade or 40 dB per range decade.

Corollary conclusions are:
(1) If the TX/Rx power budgets are unbalanced (i.e., more transmitter power at the repeater than at the terminal for equal noise figures), then reception at the repeater is the worst case and some means of discrimination will certainly be required.
(2) Use of directional antennas both at the repeater and at least on certain fixed terminals is one means of discrimination that would work.
(3) Use of separate Tx and Rx frequencies from the repeater would mitigate the terminal reception interference but not that at the repeater, i.e., it would allow use of lower repeater ERP's.

2.0 PROBABILITY OF SIMULTANEOUS INTERFERORS

Consider the desire on the part of a user terminal to transmit a packet every T seconds, where T is a relatively long time such as 2 minuits. The average rate of sending packets is then $1/T$ or λ to use the same notation as Abramson. For a packet length of τ seconds, the probability of a single particular user getting a packet through in a particular slotted packet period is approximately $\lambda\tau$ where

$$\lambda\tau = 1/12{,}000 = .833 \times 10^{-4} \text{ for a 10 millisecond packet.}$$

The exact probability is

$$P = e^{-\lambda\tau}(\lambda\tau)$$

where $e^{-\lambda\tau}$ is a number very close to unity for a single packet.

It then follows that the probability of the single user correctly getting a packet through in the large interval T is

$$P = e^{-\lambda T}(\lambda T) = 1/e \text{ where, now, } \lambda T = 1.$$

If the total population of active users, k, is equal to the number of packet intervals in the large interval T ($k = 12{,}000$), then the probability of one user getting through on average per packet is

$$P = e^{-k\lambda\tau}(k\lambda\tau) = e^{-r\tau}(r\tau) = 1/e.$$

This doesn't say how much "offered traffic" or, in effect, how many transmissions it takes. Gaarder has indicated for a slotted channel that

$r\tau = R\tau e^{-R\tau}$, where R is the sum of r desired packets and the number of retransmissions to get them through such that

$$R\tau = 1 \text{ and } r\tau = 1/e$$

stating that the "offered traffic" is 2.72 times the traffic actually passed in a "fully loaded" channel operating in a stable mode.

To assess the implication of interference during transmission, it is necessary to determine the average number of users which transmit simultaneously when the packet system is operating stabily. To see what this is, consider the following reasoning. If the radio system were "ordered" instead of random access, the probability of each of the k users getting through over the interval T would be exactly 1(assuming one user only transmits per packet). For random access, again assuming one user transmits per packet interval, the probability of getting through for each user which "comes on" during T is again unity; however only 1/e times k of the k users actually come on during T. If two users only come on randomly per packet, more of the total number, k, of the terminals would transmit over T but with less probability of getting through each time and still only k/e of the users would be served on average over T. Taking this to the limit, the average probability of a number, j, users being on simultaneously is equal in the limit to the probability of a single user getting through in a single packet, or

$$e^{-\lambda\tau}(\lambda\tau) = e^{-k\lambda\tau}(k\lambda\tau)^j/j!$$

where the form on the right is the generalization of the Poisson law where in the previous work above, j was taken equal to 1. For the parameters previously considered, j turns out to be a number like 5.

3.0 INTERFERENCE MODEL

The very nature of random access packet radio communications implies multiple stations vying simultaneously, on average, for a particular packet time slot. Perhaps, due to radio propagation conditions, there will be "preferred terminals" which will be able to get through to the repeater whenever they wish. Assuming the average data rate of each terminal like this is small, the performance of an overall segment of the packet radio network will not be altered much except to accentuate the probability of multiple simultaneous transmissions. This, an important problem to be modeled is the effect of these multiple transmissions as interference on "desired transmissions" within a particular packet period. Without much prior thought, one is compelled to view interference in terms of a "capture ratio" which is often used to characterize well disciplined analog systems where, say, a single "interferor" is vying for a channel along with another potential user. In this case, and let us go ahead and use digital detention terms, a binary decision can be made when using coherent matched-filter detection with only a small difference between the two signals either in amplitude or in phase noise. The probability of error here is governed by the varying nature of the two signals which is generally easy to bound. In packet radio, however, perhaps it is not appropriate to use the term capture ratio because it implies that to apply this ratio to an analysis, the statistics of the interference are well known and we know that this is not true. That is, we would need to know the probability that the peak interfering signal would exceed a given capture ratio and, hence, to cause a phase rotation or "click". There will, of course, be a capture effect of sorts. Let us approach the interference problem from a more conventional digital signal analysis point of view and choose a model to bound the performance which does not depend directly on the above "peaking" statistics under certain conditions.

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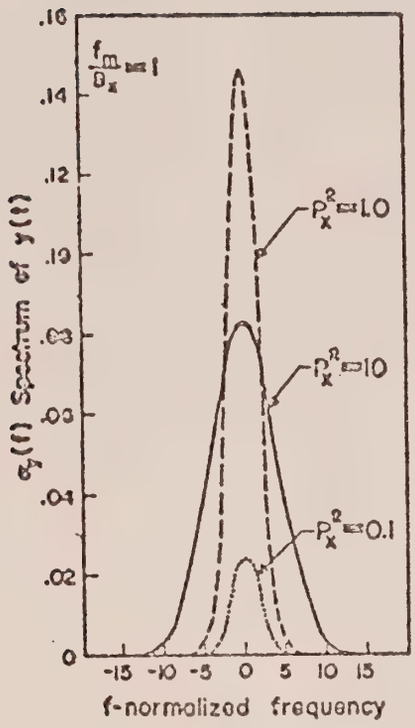
The approach assumes that the multiple interferors produce, bit by bit within a packet, normally distributed phase noise samples which modulate a stronger desired signal as seen at the output of a phase detector. The population of discreet samples is given by considering a number of interferors transmitting at least two symbol phase states with each interferor located at a generally different range from the receiver so that the phase of each will differ from another by the phase resulting from different free space propagation times interpreted as a portion of the bit period assuming bit synchronization.. Of course, the power spectral density of such a random process is defined as being the same shape as the spectrum of the desired modulation process. The important factor is that this spectrum is narrow band and the bandwidth is assumed known with respect to the center frequency of the random process which is the bit rate. The object is then to seek an interpretation of this process as increased noise on the desired signal and then to use available BER curves to define the quality of performance.

Abramson has considered the case of a carrier phase modulated by a Guassian random process under both base-band and narrow-band conditions. He gives a formula for computing the spectral density of such a modulation process which is a function of the center frequency-to-bandwidth ratio and the mean square phase deviation, P_x^2 . For the case of 1 bit/Hz of bandwidth, the above ratio is essentially unity and there is a curve for this case which is shown here as Figure 1. The spectrum shows a carrier of amplitude $e^{-P_x^2}$ in the center of a continuous spectrum of peak amplitude

$$\frac{e^{-P_x^2}}{4}(P_x^2) + \frac{e^{-P_x^2}}{8}(P_x^2)^2 + \frac{e^{-P_x^2}}{24}(P_x^2)^3 + \cdots$$

A curve of this peak amplitude vs P_x^2 is shown as Figure 2. For small deviations, this spectrum has essentially the same spectrum as the original modulation process (amplitude modulation). For larger deviations, the spectrum becomes much wider than that of the original process and takes on a Guassian shape via the central limit theorem. Let us consider the spectrum of the packet radio signal in the presence of interference to be the convolution of the data signal spectrum with the above spectrum.



$y(t) = \cos\left[\omega_c t + x(t) + \theta\right]$

B_x } = rms bandwidth of x(t)

f_m = center frequency of x(t)

$P_x^2 = \overline{x^2}$ = mean square value of x(t)

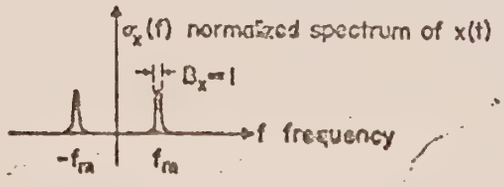


Figure 1

Normalized Spectrum of a Carrier Angle-modulated by a Narrow Band Gaussian Random Process

(A spectral line at frequency f_c of average power $e^{-P_x^2}$ is omitted from these figures)

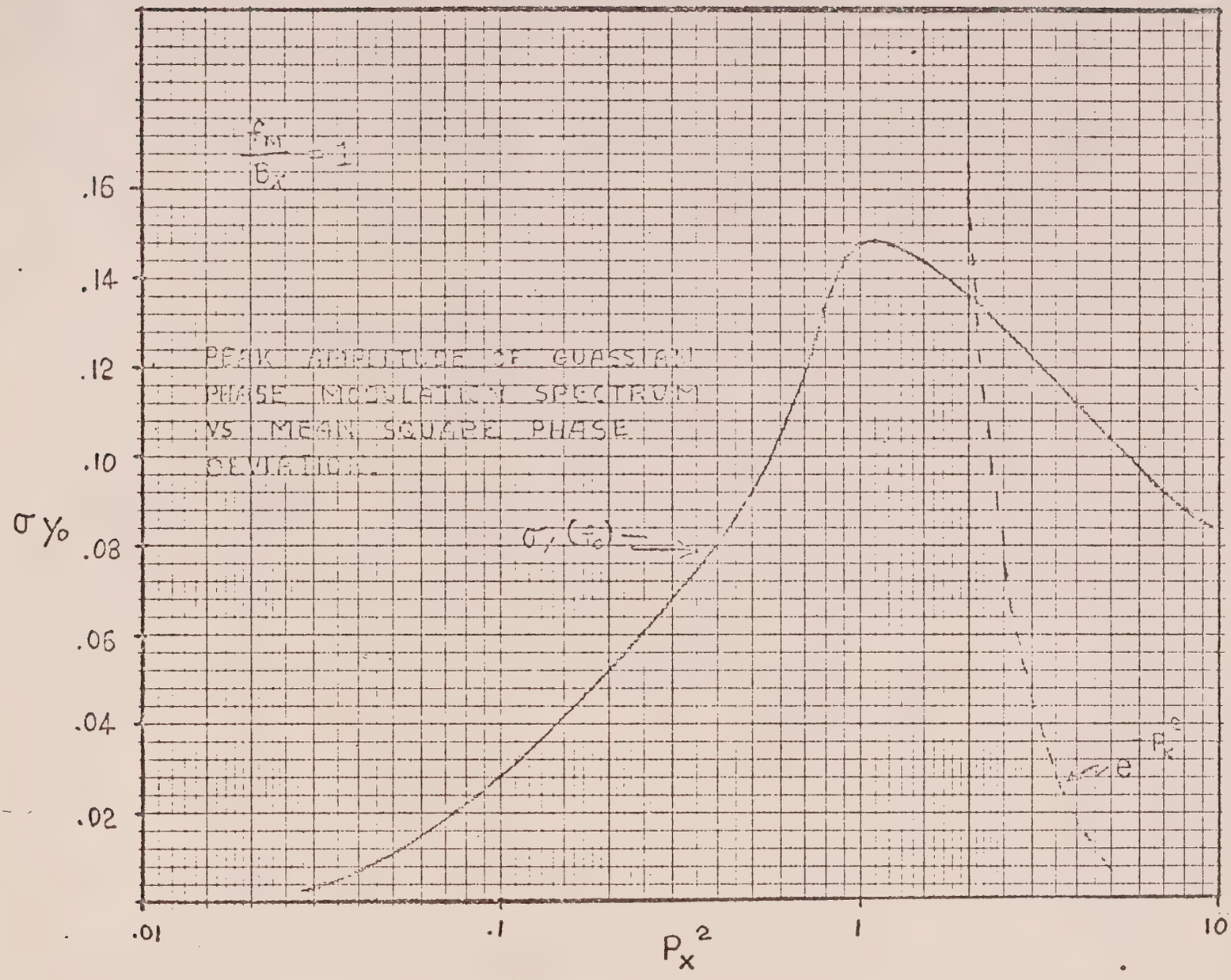

$\frac{f_M}{B_K} = \frac{1}{2}$
PEAK AMPLITUDE OF GUASSIAN
PHASE MODULATION SPECTRUM
VS MEAN SQUARE PHASE
DEVIATION.
σ_{y_0}
.16
.14
.12
.10
.08
.06
.04
.02
$\sigma_y(\tau_0) =$
$e^{-P_x^2}$
.01
.1
1
10
P_x^2

FIGURE 2

We now need to know the effect of this convolution on the peak amplitude of the continuous noise spectrum so that we can ratio the signal spectrum of peak amplitude $e^{-P^2}x$ to this noise spectrum to get a signal-to-noise ratio. If the noise spectrum is much wider than the signal spectrum, then the peak amplitude of the convolved noise spectrum is for unity signal amplitude

$$S_{*0} = B_2 S_{10}$$, where B_2 is the signal bandwidth
S_{10} is the original peak before convolving

As this bandwidth is equal to that of the interferance process and is equal to unity from the normalization of Figure 1, the convolved peak is equal to the original peak and Figures 1 & 2 apply directly for ratioing. If the resultant noise spectrum is equal in bandwidth to the signal spectrum, then the peak amplitude of the convolved spectrum is (again assuming unity amplitude for the peak of the signal spectrum)

$$S_{*0} = B_2 S_{10} \int S_{n1}\, df$$

where S_{n1} is the spectrum of the interferance normalized to both unity peak & bandwidth

It is reasonable to expect that the value of this integral will be about 1/2 so under that assuption 3 db must be added to the desired signal to noise ratio before using Figures 1 & 2 to obtain a value of P_x^2 .

The remaining problem is then to bound P_x^2 in terms of the required values of the desired signal strength and the composite interference signal strength in a radio sense. Assume that a particular terminal is receiving a signal from a repeater of rms strength B. Assume also that the terminal receives a composite interference signal of rms strength A. This is shown in the familiar "undulating" phasor triangle of angle modulation. If the repeater and terminal locations are not mobile while communicating, then B is not varying to any great degree. If we further assume that B is by design always substantially greater than A on average, then the mean phase deviation on the desired signal caused by the interferance is bounded. If the angle between B and the resultant of A & B is taken to be P_x , then a ratio of B to A can be interpreted as a signal-to-interference noise ratio.

We now need a model from which to estimate the composite interference signal, A. Consider the following: a repeater is transmitting to a particular terminal which has an omnidirectional antenna producing signal B at the receiver. At the same time, assume five terminals in the vicinity of the receiving terminal are transmitting. Assume that these transmitting terminals are uniformly distributed in area radially away from the receiving terminal. At some distance, ρ, away is the location of the closest interfering terminal. Assume that the density of terminals is given by one terminal per unit area where the unity area is $\pi\rho^2$. Call ρ the range density increment. Then at twice this distance there will be four times the area or four additional interfering repeaters. This configuration is shown geometrically in Figure 4. Now we know that in a city environment for moderate antenna heights the signal falls off as the inverse of the fourth power of range. Thus, if we assume that all these terminals at twice the range increment were in sync and transmitting the same symbol, they would produce a signal level at the receiver equal in amplitude to that from the single range increment away. Let us assume that is true so that A/2 is the signal from the interferor which is closest. This, then represents the "median peak" signal, recognizing that the assumption of five interferors is an average assumption.

We now have derived a way to relate the necessary circuit margins of the terminal to terminal and repeater to terminal to ensure a desired grade of service. It is based on the repeater to terminal signal, B, being larger significantly than the composite signal A due to interference. This, then, provides a bound on the performance of the same degree as which the traffic statistics are assumed. Any other approach requires a greater level of detail. The results with this method are optimistic in the sense that multiple simultaneous interferors producing a smaller B/A will give poorer performance. The only means of improving the situation is to use multiple frequencies, directional antennas on the terminals, or some form of spread spectrum and orthogonal coding; so, these results are sort of a baseline.

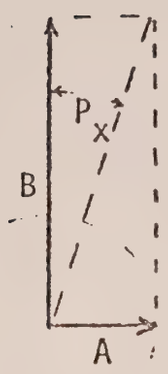


Figure 3

Familiar Phase Modulation Triangle For Two Signals

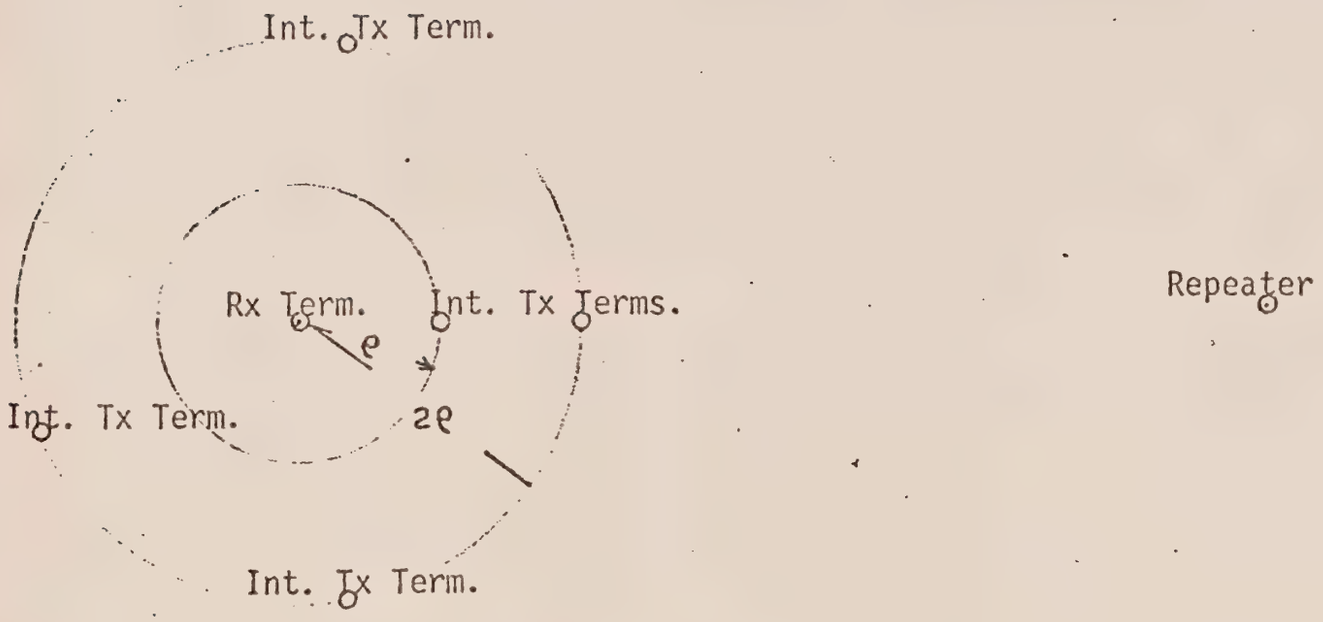


Figure 4

Geometry For Five Interferor Model

4.0 TYPICAL CALCULATION

Referring to the interference analysis, let us assume a repeater located at 5 miles range transmitting to a particular terminal; assume the frequency is 1600 MHz and the antenna height at both repeater and terminal is 30 meters. Further, assume that there are 5 interfering terminals with a range density increment of 1 mile and also that the terminal ERP is 1 watt.

The free space loss from the 1-mile interferor is 101 dB. The added loss due to the fourth power in range is 8.2 dB or a total of 109.2 dB. Assuming an equal signal from twice the range increment, the loss is 103.2 dB (figured on a peak basis) or for 1 watt ERP and 0 dB antenna gain the composite interfering signal level, A, is -103.2 dBW.

If the required S/N in a bandwidth equal to the signalling rate is taken to be 13 dB and the noise interference bandwidth is taken to be equal to that of the interfering process, then we look up at 16 dB differential from Figure 2 of the prior note, or

$$\frac{e^{-p^2} \; x}{\sigma(y)} = 40$$

This gives an rms phase deviation of 22.2 degrees which, in turn, requires that the ratio of the signal from the repeater to the composite interference, B/A, be 8.2 dB.

The signal, A, is seen to be 50 dB above thermal noise taken in a 50 dB bandwidth so the path is very definitely intererence limited. If the receiver noise figure is taken as 5.6 dB, then the signal power received from the repeater must be -97.6 + 8.2 = -89.4 dBW. The path loss from a repeater located at 5 miles range is -141.2 dB. This requires an ERP from the repeater of 51.8 dBW. As an alternate bound, working the problem for free space loss, the required ERP from the repeater is 37.4 dBW. In a city the multipath is a minimum of 2 microseconds and is probably around 4 microseconds; thus to handle about a megabit rate at 10^{-5} BER with assuming 30 dB antenna gain or about 25 kilobits without any gain requires an additional 7 dB of signal, on average, from the median. Another 7 dB or so is also

required to provide the desired grade of service, say, 90% of the time. Adding a bare minimum of 14 dB margin gives ERP's of 64.8 dBW and 51.4 dBW respectively for the two loss cases.